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### Transboundary Fishery Management

Pham Do, K.H.; Folmer, H.; Norde, H.W.

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**TRANSBOUNDARY FISHERY MANAGEMENT: A  
GAME THEORETIC APPROACH**

By Kim Hang Pham Do, Henk Folmer and Henk Norde

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Discussion paper

# Transboundary fishery management: A Game Theoretic Approach<sup>1</sup>

Kim Hang Pham Do<sup>a,b,2</sup>, Henk Folmer<sup>a,c</sup>, Henk Norde<sup>b</sup>

<sup>a</sup> *Department of Economics, and CentER, Tilburg University*

<sup>b</sup> *Department of Econometrics and Operations Research, and CentER, Tilburg University. P.O.Box 90153, 5000 LE Tilburg, the Netherlands.*

<sup>c</sup> *Department of Social Sciences, Wageningen University, the Netherlands. P.O.Box 8130, 6700 EW Wageningen*

**Abstract.** A basic issue in transboundary fishery management is the new member problem. In this paper we address the problem of allocating the profits between the charter members and the entrants, once the nations concerned have expressed an interest in achieving an agreement. Using game theory we argue that in the case of independent countries adjustment from the Nash equilibrium can be achieved by means of the proportional rule. Furthermore, we propose the population monotonic allocation scheme as management rule for division of profits within a coalition. Finally, we show that the equal division of the net gain value can be used to expand a coalition.

**Key words:** Transboundary fishery management, game theory, *Nash-Cournot* equilibrium, proportion rule, equal division, Shapley value.

## 1 Introduction

The transboundary fishery problem addressed by the U.N. intergovernmental conference from 1992 to 1995 was the escalation of high seas fisheries harvesting (OECD, 1997). The conference resulted in the 1993 U.N. Transboundary Fishery Stock Agreement<sup>3</sup>. This Agreement grants the rights of all states to utilize the fishery resource on the high seas and specifies that harvesting should be coordinated by a coalition of the traditional harvesting states<sup>4</sup>, acting through a regional or sub-regional organization, i.e. a Regional Fisheries Management

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<sup>2</sup>Corresponding author. Email address: k.h.phamdo@kub.nl

<sup>3</sup>A transboundary fish stock, also called straddling or highly migratory fish stock, is a species that can simultaneously occupy a coastal state's 200-mile Exclusive Economic Zone (EEZ) and its adjacent high seas.

<sup>4</sup>The key Article 308 of the Law of the Sea Convention (16 November 1996) states that, the Coastal states have sovereign rights over the continental shelf (the national area of the seabed) for exploring and exploiting it; the shelf can extend at least 200 miles from the shore, and more under specified circumstances.

Organization (RFMO).<sup>5</sup> The Agreement calls for those nations who wish to participate in the harvesting of the fish resource in the high seas, but are not currently members of the RFMO, to declare a willingness to join and to enter into negotiations over mutually acceptable terms of entry. However, the Agreement provides to the RFMO "no coercive enforcement powers to exclude non-member harvest or set the terms of entry into membership" (McKelvey *et al.*, 2000).

According to Munro (2000), there are two problems causing to doubt the effectiveness of RFMOs.

First, there is the so-called "interloper problem". It concerns the difficulty of controlling harvesting by non-member vessels, including individually operated vessels, but also coordinated multi-vessel "distant water fleets". Both seek targets-of-opportunity, and skim off bountiful harvests wherever they occur, but with little interest in the long term conservation of the stocks.

Second, the so-called "new member problem", which concerns the inherent difficulties of negotiating, in a timely manner<sup>6</sup>, mutually acceptable terms of entry, which will specify the petitioning nation's membership rights and obligations. Indeed, the interests of current members and of the applicants are often strongly opposed: the current members face the likelihood of having to give up a portion of their present quotas to the newcomer, and the applicant believes that it may be better off by staying outside of the coalition and continuing harvesting profitably while facing fewer constraints. Both problems arise as major issues in finding a resolution of a "just and reasonable" share of the Total Allowable Catch (TAC) under RFMO management.

Kaitala and Munro (1997) argue that the resolution of the new member problem may call for the creation of de facto property rights for the "charter members" (also called incumbent fleets or nations) of a RFMO. They raise the question of whether a possible solution might be one in which new members are required to "buy their way in" through the purchase of quota shares. The quotas allocated to "charter member" states would take the form of individual transferable quota (Munro, 2000). Thus, the charter members should become the sole beneficiaries of the fishery resource. Moreover, a potential new entrant could only access the fish stock in question by buying the fishing right and quota of an incumbent fleet. However, it is not evident that such a system based on assumptions of economic efficiency and resource sustainability is viable. It would vest substantial interests with the incumbent fleets which is likely to be strongly opposed by potential entrants.

In this paper we shall examine how a RFMO might successfully achieve effective control of a high seas fishery. We consider the transboundary fishery stock as common property and assume that all interested nations are allowed to exploit it. We also assume that all nations abide by a legally binding international con-

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<sup>5</sup>A given RFMO would be expected to have all relevant coastal states and distant water fishing nations as members.

<sup>6</sup>By "in a timely manner" the following is meant: Permit a new entrant access to a RFMO after a period over which charter members would not have to share the benefit of cooperation with new members (for further details see Kaitala and Munro, 1997).

vention under which harvesting must be sustainable and economically efficient. The potential fishing efforts and fishing costs of all nations as well as the size of the fish stock are assumed to be known. The question that we deal with in this paper is how to allocate the profits among charter members and entrants once the nations concerned have expressed an interest in achieving an agreement on sustainable and efficient exploitation of the resource. The approach taken in this paper is cooperative game theory rather than non-cooperative game which is common in environmental and resource economics (see among others, Folmer *et al.*, 1998 and the references therein). Whereas in non-cooperative game theory the emphasis is on self-interested behaviour within a given set of rules, cooperative game theory starts from the assumption that players have committed themselves to a binding agreement. The emphasis is on the allocation of payoffs to achieve stable coalitions (see, for example, Moulin, 1995).

As described above, the main objective of a RFMO is the management of a resource for a set or coalition of countries that have expressed an interest in sustainable and efficient harvesting. This implies that cooperative game theory is more appropriate to analyse the research question of this paper than non-cooperative game approach. We will show that both charter members and potential entrants can gain from management rules regardless waiting time and transferable membership rights.<sup>7</sup>

In section 2 the fishery resource management problem is set up as an oligopoly game. Section 3 introduces notations and definitions needed for the game theoretic analysis of the noncooperative and cooperative solutions in section 4. Section 5 focuses on various allocation schemes under entrance. Conclusions follow in section 6.

## 2 The fishery problem

Assume that the demand for fish and the supply of "fishing effort" (combined services of labor and capital devoted to harvesting) are both perfectly elastic. The exploitation of a fishery resource can then be described by the following differential equation

$$\dot{x} = F(x) - H(E, x) \quad (2.1)$$

where  $x$  is a non-negative state variable representing the fishery resource or biomass at time  $t$ ;  $F(x)$  is a *growth function* of biomass satisfying  $F(0) = F(b) = 0$ , and  $F''(x) \leq 0$  for  $x \in (0, b)$ . Here  $b$  denotes the carrying capacity of the resource (i.e., *natural capacity*);  $E$  is fishing effort and  $H(E, x)$  is the harvesting or production function<sup>8</sup>.

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<sup>7</sup>We shall not address the interloper problem in this paper.

<sup>8</sup>The harvesting function is often assumed to be *bilinear* in the stock,  $x$ , and the fishing effort,  $E$ , such that  $H(x, E) = qEx$  or  $H(t) = qE(t)x(t)$ , where  $q$  is the catchability coefficient (see for example Clark, 1990).

Resource growth is affected by the stock and harvesting. Hence, if  $E$  is constant, then the stock evolves towards the *natural equilibrium*  $x = x_b(E)$  defined by

$$F(x_b(E)) - H(E, x_b(E)) = 0, \text{ i.e. } \dot{x} = 0.$$

We assume for simplicity that  $F(x) = x(b - x)$  and  $H(E, x) = Ex$ . In that case the natural equilibrium is  $x_b(E) = b - E$ , if  $b \geq E$ . If  $E > b$  the stock decays rapidly (Clark, 1990). Therefore, to avoid depletion of a fish resource,  $E$  should be less than  $b$ . Under these circumstances, we can assume that the fish stock remains in equilibrium,  $x_b(E)$ . The production function is now determined by

$$Y := H(E, x) = E(b - E), \text{ for } 0 \leq E \leq b \quad (2.2)$$

and called the *yield-to-effort* equation. The economic *payoff* to harvesting effort (or *economic rent*) is

$$\pi(E) = pY - cE = pE(b - E) - cE = pE[(b - c/p) - E] \quad (2.3)$$

where  $p$  is the unit price of harvest landed and  $c$  is the unit cost of effort. If we normalize so that  $p = 1$ , then the payoff function is

$$\pi(E) = Y - cE = E(b - c - E), \text{ for } 0 \leq E \leq b \quad (2.4)$$

and harvest is profitable only when  $0 < E < b - c$ .

Under centralized (i.e. monopolistic) management by the RFMO, the effort level  $E$  which determines the harvest or total allowable catch can be derived from the equation (2.4).<sup>9</sup>

Now suppose that, instead of centralized management, there are  $N$  countries (players),  $N = \{1, 2, \dots, n\}$ , independently harvesting the fish stock simultaneously and each country has a fishing effort level (or fishing fleet of size)  $E_i$ . The yield-to-effort equation is unchanged (given  $b$ ) but the total effort now is the sum  $E = \sum_{i=1}^n E_i$ , and the equilibrium of fish stock is  $x_b(E) = b - \sum_{i=1}^n E_i$ . Under these circumstances, the natural equilibrium can be considered as the value of the fish resource under exploitation of  $N$  countries.<sup>10</sup> If the individual country payoffs are proportional to the corresponding fishing effort levels and if each country has its own cost function (i.e. harvest depends on the unit of fishing cost  $c_k$ ), then country  $k$ 's payoff can be written as

$$\pi_k(E_1, \dots, E_N) = p(E)E_k - c_k E_k \quad (2.5)$$

where  $p(E) = b - \sum_{i=1}^n E_i$  is the (natural) price.

Equation (2.5) shows that each country's payoff depends on the aggregate effort and on the country's own effort. It captures the fact that for a country to

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<sup>9</sup>  $\max_E \pi(E) = \max_E \{E(b - c - E)\} = \frac{(b-c)^2}{4}$  at  $E = \frac{b-c}{2}$

<sup>10</sup> The linear function  $p(E) = b - \sum_{i=1}^n E_i$  can be seen as the linear inverted supply curve in the Cournot situation with  $n$  producers and  $\sum_{i=1}^n E_i$  is total (competitive) quantity.

maximize its payoff, it will calculate its optimal effort level taking into account the anticipated effort level by its opponents. As a consequence, given a fishing unit cost, the fish stock declines or increases, depending on whether the marginal product increases or declines. In the remainder of the paper, we will use  $X_k = [0, l_k]$  to denote the strategy set, i.e. effort levels, of country  $k$ , where  $l_k$  is the maximum effort level of country  $k$ .

The fishery situation can now be formulated as  $\mathfrak{F} = (N, X, C, \pi, b)$  where

- (i)  $N$  is the finite set of countries (*players*);
- (ii)  $X = \prod_{k=1}^n X_k$ , where  $X_k$  denotes the set of (harvesting) effort levels (*strategy set*) of country  $k$ ;
- (iii)  $C = (c_k)_{k \in N}$ , where  $c_k$  is the (constant) marginal cost of country  $k$ ;
- (iv)  $\pi = (\pi_k)_{k \in N}$ , where  $\pi_k$  is the profit (*payoff*) function of country  $k$ ; and
- (v)  $b$  is the natural capacity of the resource.

The tuple  $\mathfrak{F} = (N, X, C, \pi, b)$  described above is called a *fishery problem*.

### 3 Preliminaries

For each  $k \in N$  and for every non-empty subset  $S \subseteq N$ , we define

$$X = \prod_{k=1}^n X_k, \quad X_S = \prod_{j \in S} X_j, \quad \text{and} \quad X_{-S} = X_{N \setminus S} = \prod_{j \notin S} X_j.$$

Occasionally, notations like  $(x_i, x_{-i})$  or  $(x_S, x_{N \setminus S})$  are used if the strategy of player  $i$  or coalition  $S$  needs stressing, where, as usual,  $x_{-i}$  denotes the vector  $x$  with the  $i$ th component deleted. The sum  $\sum_{i \in S} x_i$  is denoted by  $x(S)$  for every  $S \subseteq N$ . In addition, for every real number  $a \in \mathfrak{R}$ , we define  $a_+ = \max\{a, 0\}$ .

**Definition 3.1** *A fishery game (FG) is an  $n$ -person game*

$$\Gamma = \langle X_1, \dots, X_n; \pi_1, \dots, \pi_n \rangle, \quad \text{where}$$

- (i)  $X_k = [0, l_k]$  is the strategy set of player  $k$ , and  $0 < l_k < \infty$ .
- (ii)  $\pi_k(x) = p(x)x_k - c_k x_k$  is the payoff function of player  $k$ , where  $p(x) = (b - \sum_{j=1}^n x_j)_+$  and  $c_k > 0$ .

The payoff for the  $k$ th player depends on the strategies of all the other players as well as on his own strategy. It is easy to see that for  $x \in X$ ,  $\pi_k(x)$  is continuous in  $x$ .

**Definition 3.2** *A vector  $x^* = (x_k^*)_{k \in N} \in X$  is a (Nash) equilibrium if for every  $k \in N$  and  $y_k \in X_k$ ,  $\pi_k(x^*) \geq \pi_k(y_k, x_{-k}^*)$ , i.e. player  $k$  has no incentive to deviate from  $x_k^*$  when all other players play  $x_{-k}^*$ .*

To design behavioral patterns, where explicit communication can take place, we assume that the countries form coalitions. The objective of each country in the coalition is to maximize total profit for the coalition that the countries of  $S$  can

bring about by their cooperation. The structural inefficiency of the competitive (non-cooperative) equilibrium is then interpreted as an incentive to cooperate.

**Definition 3.3** For an  $n$ -player FG and a coalition structure<sup>11</sup>  $\kappa$  of  $N$ , we define the strategy set  $X_S$  of each coalition  $S \in \kappa$  as the Cartesian product of the strategy sets of the players belonging to  $S$ . The coalition's payoff function equals the sum of payoff functions of players belonging to  $S$ . That is, with  $(x_S, x_{-S}) \in X$ ,

$$\pi_S(x) = \sum_{j \in S} \pi_j(x) = \sum_{j \in S} [p(x_S, x_{-S})x_j - c_j x_j]. \quad (3.2)$$

**Definition 3.4** A cooperative (profit) game is an ordered pair  $(N, v)$  where  $N$  is the set of players, and  $v: 2^N \rightarrow \mathbb{R}$  is the characteristic function relating each coalition  $S \subseteq N$  to a real number  $v(S)$ , representing the total payoff (profit) which  $S$  is able to generate through internal cooperation, with the convention that  $v(\emptyset) = 0$ .

For every cooperative game  $(N, v)$  and all  $T \subseteq N$  the subgame  $(T, v|_T)$  is defined by  $v|_T(R) = v(R)$  for all  $R \subseteq T$ . A payoff vector  $(x_1, \dots, x_n)$  of a cooperative game  $(N, v)$  is an  $n$ -dimension vector describing the payoffs to the players, where player  $i \in N$  receives  $x_i$ .

The central stability concept in cooperative game theory is the *core*,  $C(N, v)$ , defined as follows.

**Definition 3.5** For a cooperative game  $(N, v)$ , a payoff vector  $(x_i)_{i \in N}$  is in the core,  $C(N, v)$ , if

$$\sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \subset N.$$

The first part of this definition ensures that the payoff vector is *feasible* (the so-called *efficiency* condition) for the *grand coalition*  $N$ . The second part introduces a *stability* requirement which states that no subcoalition  $S$  by acting on its own can achieve an aggregate payoff which is higher than the share that it receives under payoff vector  $x$ . If we take for  $S$  the singleton sets we get the *individual rationality* requirement, stating that every player should receive at least this stand alone value.

Therefore, once an allocation from the core has been selected, no coalition on its own can improve the payoff of all its members. However, the core of a cooperative game may be empty. A standard procedure to check whether a game  $(N, v)$  has a non-empty core arises from the *Bondareva-Shapley theorem* (cf. Bondareva, 1963 and Shapley, 1967). This theorem indicates that the necessary and sufficient conditions for a game to have a non-empty core are the balancedness conditions.

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<sup>11</sup>A coalition structure  $\kappa$  is a partition of the set  $N$  of players, i.e.  $\kappa = \{S_1, S_2, \dots, S_k\}$ ; it describes which coalitions are formed and coexist.



**Definition 3.6** A cooperative game  $(N, v)$  is balanced if for every non-negative vector of weights  $(\lambda_S)_{S \subset N, S \neq N}$ , which satisfies  $\sum_{S: i \in S} \lambda_S = 1$  for every  $i \in N$ , we have  $v(N) \geq \sum_{S \subset N, S \neq N} \lambda_S v(S)$ .

The balancedness condition states that there is no feasible pattern of coalition formation that yields a higher aggregate payoff than the grand coalition can achieve.

**Definition 3.7** A cooperative game  $(N, v)$  is called convex if it satisfies increasing returns with respect to the coalition size, i.e. if for every  $S, T \subset N$  and every  $i \in N$  such that  $S \subset T \subseteq N \setminus \{i\}$ , it follows that

$$v(T \cup \{i\}) - v(T) \geq v(S \cup \{i\}) - v(S) \quad (3.3)$$

The term  $v(S \cup \{i\}) - v(S)$  is interpreted as the marginal worth of player  $i$  to the coalition  $S$ . Hence, convexity implies that the larger a coalition becomes, the greater is the marginal contribution of new members.

## 4 Analysis

To analyze the competitive and cooperative outcome of a fishery game, we first present some general results and properties. We begin with the following definition.

**Definition 4.1** For every fishery game  $(FG)$ , the conservation strategy space,  $X^*$ , is

$$X^* = \{x = (x_1, \dots, x_n) \in X \mid \sum_{k=1}^n x_k \leq b\}$$

**Proposition 4.1** For every  $FG$ , there is no equilibrium in  $X \setminus X^*$ . That is, an equilibrium belongs to  $X^*$  only.

**Proof.** Suppose that there exists an equilibrium  $x^* \notin X^*$ , then  $p(x^*) = (b - \sum_{k=1}^n x_k^*)_+ = 0$ . Since  $\sum_{k=1}^n x_k^* = x^*(N) > b$ , the vector  $x^*$  has at least one positive element, which we denote  $x_m^*$ . Hence,  $\pi_m(x_m^*, x_{-m}^*) = -c_m x_m^* < 0$ , and the best strategy of player  $m$  should be  $y_m^* = 0$ . This contradicts the assumption that  $x^*$  is an equilibrium point. ■

### 4.1 Competitive or noncooperative outcome

In order to discuss the competitive outcome of a fishery game we will first examine the reaction set of each player in greater detail.

**Definition 4.2** For every  $x_{-k} \in X_{-k}$ , player  $k$ 's rational reaction set is defined

$$as \ R_k(x_{-k}) = \{y_k \in X_k | \pi_k(y_k, x_{-k}) = \max_{z_k \in X_k} \pi_k(z_k, x_{-k})\}.$$

For every  $FG$  each strategy set is a compact subset of the real line  $\mathfrak{R}$  and the payoff function of each player is continuous in the action of all players as well as in its own action. Those are sufficient conditions to ensure that the rational reaction sets  $R_k(x_{-k})$  are not empty, for all  $k \in N$  and  $x_{-k} \in X_{-k}$ . In order to determine the reaction set  $R_k(x_{-k})$  player  $k$  has to maximize the function  $t_k \rightarrow \pi_k(t_k, x_{-k}) = (b - \overline{x_{-k}} - t_k)_+ t_k - c_k t_k$  on his strategy set  $X_k = [0, l_k]$ . Here  $\overline{x_{-k}}$  denotes the sum  $\sum_{j \neq k} x_j$ . The graph of this function is depicted in Figure 1.

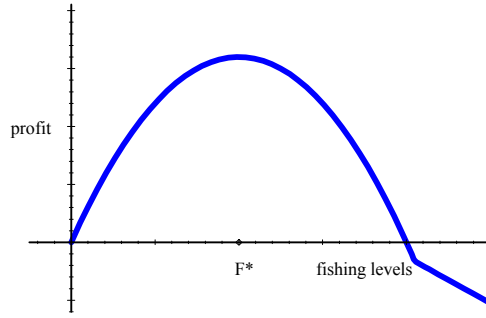


Figure 1. The payoff function of player  $k$ , where  $F^* = \frac{b - \overline{x_{-k}} - c_k}{2}$ .

From this graph it is easy to get the following observations:

- (1) If  $b - \overline{x_{-k}} - c_k \leq 0$  then player  $k$  will decide to take  $y_k = 0$ ;
- (2) If  $0 < \frac{b - \overline{x_{-k}} - c_k}{2} \leq l_k$ , then the optimal strategy of player  $k$  is  $y_k = \frac{1}{2}(b - \overline{x_{-k}} - c_k) \in (0, l_k)$ ;
- (3) If  $\frac{b - \overline{x_{-k}} - c_k}{2} > l_k$  then the optimal strategy of player  $k$  is  $y_k = l_k$ .

The following lemma and corollary are obtained.

**Lemma 4.1** *For every  $k \in N$  and  $x_{-k} \in X_{-k}$ , the reaction of player  $k$  is a singleton set. That is*

$$R_k(x_{-k}) = \begin{cases} \{0\} & \text{if } b - \overline{x_{-k}} \leq c_k \\ \{\frac{b - \overline{x_{-k}} - c_k}{2}\} & \text{if } c_k < b - \overline{x_{-k}} \leq c_k + 2l_k \\ \{l_k\} & \text{if } b - \overline{x_{-k}} > c_k + 2l_k \end{cases}$$

Figure 2 depicts the reaction set of player  $k$  as function of  $\overline{x_{-k}}$ , i.e., the total effort of the other players. Note that this function is non increasing.

**Corollary 4.1** *The best response of player  $k$ ,  $R_k : X \rightarrow \mathfrak{R}$ , is a non increasing function of  $x$ .<sup>12</sup>*

<sup>12</sup>We say  $x \leq y$  if  $x_i \leq y_i, \forall i \in N$ ; and  $x < y$  if  $x_i < y_i, \forall i \in N$ .

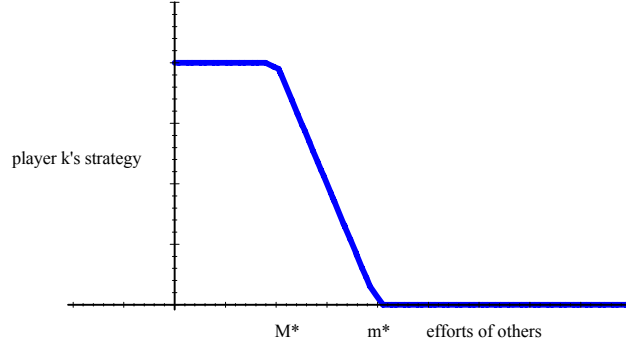


Figure 2. The reaction function of player  $k$ , where  $M^* = b - c_k - 2l_k$  and  $m^* = b - c_k$ .

For all  $k \in N$  and  $x_{-k} \in X_{-k}$  the reaction set  $R_k(x_{-k})$  is a singleton set. Following a quadratic programming approach (see Mañas, 1972) one can prove that a fishery game has a unique Nash equilibrium.

**Theorem 1** *A fishery game  $FG$  has a unique Nash-Cournot equilibrium*

**Remark 4.1** *The outcome of the non-cooperative game is virtually identical to that of the unregulated open access fishery, particularly when combined harvesting efforts in the Nash equilibrium is larger than  $b/2$ .<sup>13</sup>*

## 4.2 Cooperative outcome

For each fishery problem a related cooperative game  $(N, v)$  can be defined by means of the  $\alpha$ - and  $\beta$ - conversions introduced by Aumann (1959).

The  $\alpha$ -characteristic function of a fishery game  $\Gamma$  is the function  $v_\alpha$  defined by

$$v_\alpha(S) = \text{Max}_{x_S \in X_S} \text{Min}_{x_{-S} \in X_{-S}} \pi_S(x_S, x_{-S}) \quad (4.1)$$

whereas the  $\beta$ -characteristic function of the fishery game  $\Gamma$  is the function  $v_\beta$  defined by

$$v_\beta(S) = \text{Min}_{x_{-S} \in X_{-S}} \text{Max}_{x_S \in X_S} \pi_S(x_S, x_{-S}) \quad (4.2)$$

The  $\alpha$ -characteristic function represents a *prudent perception* by the members of the coalition  $S$  about their capability to guarantee themselves the payoff  $v_\alpha(S)$  if they choose the joint strategy  $x_S$  *before* the joint strategy  $x_{-S}$  of the opposition  $N \setminus S$  has been chosen, *i.e.* coalition  $S$  can ensure to its members the maximum (total) payoff while choosing the strategy combination  $x_S$  regardless of what the opposition  $N \setminus S$  does.

The  $\beta$ -characteristic function represents an *optimistic perception* by the members of the coalition  $S$  in the sense that the opposition  $N \setminus S$  can prevent that the players in  $S$  get more than  $v_\beta(S)$ . Therefore, in the  $\alpha$ - framework, a coalition

<sup>13</sup>This is due to the fact that at  $b/2$  the yield of the fish resource is maximally sustainable.

$S$  obtains the payoff it can guarantee itself, irrespective of the strategy choice of the players in  $N \setminus S$ , whereas in the  $\beta$ - framework, the coalition  $S$  obtains the maximum payoff from which it can not be prevented by the players in  $N \setminus S$ . It is, of course, easy to see that

$$v_\alpha(S) \leq v_\beta(S) \text{ for all } S \subseteq N \text{ and } v_\alpha(N) = v_\beta(N).$$

Moreover, the payoffs defined by (4.1) and (4.2) coincide for every given fishery problem, i.e.  $v_\alpha(S) = v_\beta(S)$  for all  $S \subseteq N$  (see Norde *et al.*, 2000). This implies that for every coalition the amount which this coalition can guarantee itself, and the maximum amount from which they can not be prevented by the opposition are the same.

**Example 4.1** Consider the FG with  $N = \{1, 2, 3\}$ ,  $l_1 = 14$ ,  $l_2 = 8$ ,  $l_3 = 12$ ,  $c_1 = 2$ ,  $c_2 = 4$ ,  $c_3 = 16$ , and  $b = 60$ . Norde *et al.* (2000) provided a formula for calculation of the value  $v_\alpha(S)$  ( $= v_\beta(S)$ ). For the sake of completeness, it has been included in the Appendix. Applying this formula, we obtain  $v_\alpha(123) = 776$ ;  $v_\alpha(12) = 512$ ;  $v_\alpha(13) = 520$ ;  $v_\alpha(23) = 321$ ;  $v_\alpha(1) = 336$ ;  $v_\alpha(2) = 176$ ;  $v_\alpha(3) = 121$ .

We now turn to some important characteristics of cooperative fishery games (CFGs). First of all, the convexity of a CFG follows directly from Theorem 1 in Norde *et al.* (2000).<sup>14</sup>

**Proposition 4.2** *Every cooperative fishery game (CFG) is a convex game.*

As a CFG is convex, the following corollaries hold.

**Corollary 4.2** *A CFG has a non empty core, i.e. a CFG is a balanced game.*

**Corollary 4.3** *A CFG is zero-monotonic, i.e. for every  $S, T$  such that  $S \subseteq T$ ,  $v(S) - \sum_{k \in S} v(\{k\}) \leq v(T) - \sum_{k \in T} v(\{k\})$ .*

**Definition 4.3** *Let  $(N, v)$  be a CFG. We say that an agreement can be achieved if there exists an allocation  $x$  such that  $x$  is individually rational and efficient, i.e.  $x_i \geq v(\{i\})$  for all  $i \in N$  and  $\sum_{i=1}^N x_i = v(N)$ .*

**Theorem 2** *The Shapley value<sup>15</sup> of a CFG is a solution that is both individually rational and efficient. Moreover, the Shapley value is in the midpoint of the core of the CFG.*

**Proof.** Follows from Shapley's theorem (cf. Shapley, 1953) and convexity of  $(N, v)$ . ■

In the next section we will adopt the Shapley value to develop management rules for a RFMO.

<sup>14</sup>The analysis in this section is based on oligopoly games without transferable technologies, as modeled by Norde *et al.* (2000).

<sup>15</sup>The Shapley value,  $\Phi(v) = (\Phi_k(v))_{k \in N}$ , of a cooperative game  $(N, v)$ , is defined by  $\Phi_k(v) = \sum_{S \subseteq N \setminus \{k\}} \frac{|S|!(n-1-|S|)!}{n!} (v(S \cup \{k\}) - v(S))$ . Roughly speaking, the Shapley value means that each player should be paid according to how valuable his/her cooperation is for the other players (for example, see Friedman, 1990).

## 5 Implications for RMFO management

In this section we are applying the theoretical results of section 4 to three typical management problems that RFMOs encounter. The first problem relates to the allocation of the payoff for a set of countries that have agreed on the reduction of the fishing effort from the competitive equilibrium so as to prevent the stock from extinction or to increase the profit level. However, the countries have not formed a coalition that operates collectively to achieve sustainable and efficient harvesting. Hence, the individual countries act independently. The second problem concerns the allocation of the payoff within a coalition. We do not only consider the grand coalition but coalitions of all possible sizes, the individual country or single country coalition being the smallest. The third problem relates to the expansion of a coalition. In this regard we consider the direct expansion of single country coalitions to the grand coalition as well as the stepwise expansion of original country coalitions to the grand coalition as examples. Before going into detail we make the following observations.

- (i) We consider charter members as a coalition  $S$ , whereas the set of outsiders of  $S$ , *i.e.*  $N \setminus S$ , consists of all potential entrants.
- (ii) We adopt the following related conditions for an agreement to be self-enforcing (Botteon and Carraro, 1997). First, the coalition must be profitable, that is, each player gains from joining the coalition relative to its position when there is no cooperation. Second, the coalition must be stable, that is, no player should have an incentive to deviate from its coalition.
- (iii) Consider  $n$  players ( $n \geq 2$ ) that interact in a transboundary fish stock. Let  $(P_j(S))_{S \subseteq N, j \in S}$  be an *allocation scheme*<sup>16</sup>, where  $P_j(S)$  is the payoff of player  $j$  in a coalition  $S$ , and  $P_j(S \cup \{k\})$  is the payoff of player  $j$  when coalition  $S$  is expanded by adding player  $k \notin S$  to the coalition. We assume that a coalition can be expanded if (a) there exists an allocation scheme such that  $P_j(S \cup \{k\}) - P_j(S) \geq 0$  for all  $j \in S$  and (b) countries belonging to the coalition are committed to cooperation.
- (iv) We assume that all countries are interested in the preservation of the stock.
- (v) Let  $c_i$  denote the fishing unit cost of country  $i$ . We assume that all costs are ordered in the following way:  $c_1 \leq c_2 \leq \dots \leq c_n$ .

We now consider the three management cases for the RFMO distinguished above.

### 5.1 Independent players: The proportional rule

As described in section 2, a fish stock will be depleted if total (fishing) effort exceeds the carrying capacity of the stock  $b$ . In a similar vein, if total (fishing) effort exceeds  $b/2$ , total profit is smaller than the maximum profit. The management problem comes down to the reduction of the harvesting level. For

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<sup>16</sup> An *allocation scheme* is a *payoff scheme* that does not only provide a payoff vector for a specific game but also for all its subgames.

that purpose we make use of bankruptcy analysis <sup>17</sup> (for example, see Thomson, 1995).

Let  $(x_i^*)_{i \in N}$  be a competitive equilibrium. Define the *proportional rule*,  $PROP(i)$ , applied to every player  $i$  such that

$$PROP(i) = x_i^* \left( \frac{M}{x^*(N)} \right), \quad (5.1)$$

where  $M = b/2$ .

**Proposition 5.1** *The payoff under the proportional rule is larger than the competitive payoff for every player*<sup>18</sup>.

**Proof.** Let  $x^*$  be the competitive equilibrium, i.e. the  $NE$ , for which  $\frac{b}{2} < x^*(N) \leq b$ . Let  $x^A = (x_i^A)_{i \in N}$  be the adjustment strategy from  $x^*$  by the  $PROP$  rule. That is  $x_i^A = x_i^* \left( \frac{b/2}{x^*(N)} \right) \leq x_i^*$ , and  $x^A(N) = \frac{b}{2}$ .

Denote  $\pi_i(x^*)$  and  $\pi_i(x^A)$  as the competitive and  $PROP$  payoffs for each player  $i \in N$ , respectively. Define  $\Omega_i = \pi_i(x^*) - \pi_i(x^A)$ .

It is sufficient to prove that  $\Omega_i \leq 0$ . If  $x_i^* = 0$  then  $x_i^A = 0$  and clearly  $\Omega_i = 0$ . Now assume that  $x_i^* > 0$ . We have  $\Omega_i = [b - x^*(N) - c_i]x_i^* - [b - x^A(N) - c_i]x_i^A = (b - c_i)(x_i^* - x_i^A) - [x^*(N)x_i^* - x^A(N)x_i^A]$ , and  $x^*(N) = \frac{b}{2} \frac{x_i^*}{x_i^A}$ . Therefore

$$\begin{aligned} \Omega_i &= (x_i^* - x_i^A)(b - c_i) - \frac{b}{2x_i^A} [(x_i^*)^2 - (x_i^A)^2] = \\ &= (x_i^* - x_i^A) \left[ (b - c_i) - \frac{b}{2x_i^A} (x_i^* + x_i^A) \right] \leq -c_i(x_i^* - x_i^A) < 0 \quad \blacksquare \end{aligned}$$

**Example 5.1.** Consider the 2-person FG in which  $b = 30$ ,  $y_1 = 18$ ,  $y_2 = 16$ ,  $c_1 = 4$ , and  $c_2 = 5$ .

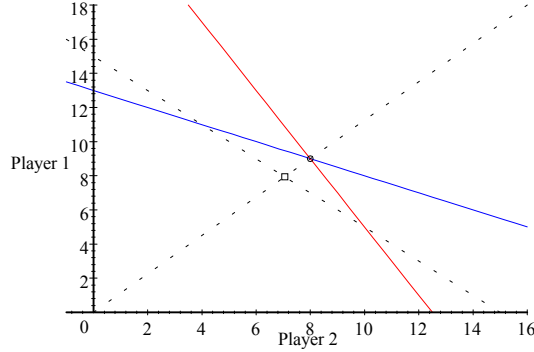


Figure 3. Best replies (solid lines) and adjusted effort levels (dotted lines).

<sup>17</sup>A bankruptcy problem is a triple  $(N, E; d)$ , where  $N$  is the finite set of players,  $E \in (0, \infty)$  is the state which has to be divided and  $d = (d_1, \dots, d_N)$  is the vector of player claims such that  $d(N) = \sum_{i=1}^N d_i \geq E$ .

<sup>18</sup>One can prove Proposition 5.1 in the case  $M$  is reduced as long as the following condition holds:  $\frac{b-c_i}{2} < M$ , for all  $i \in N$ .

The competitive equilibrium  $x^* = (9, 8)$  which is the intersection point of the two solid lines in Figure 3. Since  $\bar{x}^* = 17 > \frac{b}{2} = 15$  we have a suboptimal profit  $\pi(x^*)$ . The adjustment of the  $NE$  according to the propotional rule leads to  $x^A = (9, 8) * \frac{15}{17} = (7.94, 7.06)$ , which is intersection point of the two dotted lines in Figure 3. The payoff,  $\pi(x^A) = (87.35, 70.59)$  is larger than  $\pi(x^*) = (81, 64)$ .

## 5.2 Management within a coalition: The population monotonic allocation scheme

We start by analysing which conditions upon an allocation scheme,  $(P_j(S))_{S \subseteq N, j \in S}$ , have to be satisfied in order to induce an additional country to enter a stable coalition. If a coalition  $S$  is stable and if its members are committed to cooperation, the additional benefit of player  $j$  when country  $k$  enters  $S$  is  $P_j(S \cup \{k\}) - P_j(S)$ . A new entrant  $k$  can be accepted to join the coalition  $S$  if it does not harm any members of  $S$ , that is,  $P_j(S \cup \{k\}) - P_j(S) \geq 0$  for all  $j \in S$ . Therefore, in order to deal with entrants, one should have an allocation scheme,  $(P_j(S))_{S \subseteq N, j \in S}$ , satisfying this monotonicity property. Such an allocation scheme can be considered acceptable for every player.

Sprumont (1990) introduced a concept that exhibits this monotonicity property. It guarantees that once a coalition is formed, no player in this coalition has an incentive to form a smaller coalition, since the payoff of any player increases as the coalition he belongs to grows larger. An allocation scheme that satisfies this property as well as the property of efficiency for all subgames is called a *population monotonic allocation scheme*.

**Definition 5.1** A vector  $(x_{i,S})_{S \subseteq N, i \in S}$  is a *population monotonic allocation scheme (PMAS)* for the cooperative game  $(N, v)$  if it satisfies the following conditions:

- (i)  $\sum_{i \in S} x_{i,S} = v(S)$  for all  $S \subseteq N$
- (ii)  $x_{i,S} \leq x_{i,T}$  for all  $S, T \subseteq N$  with  $S \subseteq T$  and all  $i \in S$ .

Sprumont (1990) also showed that convex games have a *PMAS*. In fact, he proved that the Shapley value for the at large game and each subgame provides a *PMAS*. The following theorem is a direct consequence of Sprumont's observation and Proposition 4.2 which states that every fishery game is a convex game.

**Theorem 3** *Fishery games have a PMAS. Furthermore, the Shapley values calculated for each subgame give the PMAS.*

**Example 5.2** Consider the FG in example 4.1.

The marginal contributions of the players for a given order, say 1-2-3, are obtained in the following way. Player 1 is given his stand alone value  $v(\{1\}) = 336$ , player 2 is given his marginal contribution to coalition  $\{1, 2\}$ , i.e.  $v(\{1, 2\}) - v(\{1\}) = 176$ , and player 3 is given his marginal contribution to coalition  $\{1, 2, 3\}$ , i.e.  $v(\{1, 2, 3\}) - v(\{1, 2\}) = 264$ . The payoffs for the various orders of joining are

<i>order</i>	marginal contribution of player 1	marginal contribution of player 2	marginal contribution of player 3
1-2-3	336	176	265
1-3-2	336	256	184
2-1-3	455	176	145
2-3-1	336	176	264
3-2-1	455	200	121
3-1-2	399	256	121
Shapley value	$386\frac{1}{6}$	$206\frac{4}{6}$	$183\frac{1}{6}$

For any of the six orders we can compute a marginal vector and the average of these vectors is the Shapley value. Computing the Shapley value for every subgame we get the following *PMAS* :

	1	2	3
123	$386\frac{1}{6}$	$206\frac{4}{6}$	$183\frac{1}{6}$
12	336	176	*
13	$367\frac{1}{2}$	*	$152\frac{1}{2}$
23	*	188	133
1	336	*	*
2	*	176	*
3	*	*	121

### 5.3 Expansion of coalitions: Equal division of the net gain value

Consider coalitions  $S$  and  $K$  that decide to merge. A merger is stable if neither  $S$  nor  $K$  is harmed, in the sense that neither receives a lower payoff after the merger than before. We define the *net gain value* of  $S$  and  $K$ ,  $ngv(S, K) = v(S \cup K) - (v(S) + v(K))$ . Neither  $S$  nor  $K$  will be harmed by the merger if  $ngv(S, K)$  is nonnegative, and divided equally among the members in  $S \cup K$ . Below the net gain value is used to analyse the direct expansion of single country coalitions to the grand coalition as well as the stepwise expansion from formed coalitions to the grand coalition.

#### 5.3.1 1-country (single) coalitions<sup>19</sup> vs the grand coalition

Let  $v(\{i\})$  be the value of the singleton coalition  $\{i\}$  and let the *net returns to the grand coalition*,  $nrs(N)$ , be defined by  $nrs(N) = v(N) - \sum_{i \in N} v(\{i\})$ . The *equally gain payoff* is obtained by adding  $\frac{nrs(N)}{N}$  to the value  $v(\{j\})$  of each player such that  $P_j(N) = \frac{nrs(N)}{N} + v(\{j\})$ . By convexity of the *CFG*, the net return is non negative. Therefore, the payoff of country  $j$ ,  $P_j(N) \geq v(\{j\})$ . Hence, the following proposition applies.

<sup>19</sup>Note that under the preservation assumption,  $v(\{i\})$  now can be considered as an individual (rational) coalition.



**Proposition 5.2** *The payoff for each player increases under the equally gain payoff.*

**Example 5.3** Consider the FG in example 4.1.

The unique competitive equilibrium for this FG is (14,8,11) with associated payoffs (350, 184, 121). The equilibrium is adjusted to (14,8,11)  $(\frac{30}{33}) = (12.73, 7.27, 10)$ , and the payoffs are  $(356\frac{4}{11}, 189\frac{1}{11}, 140)$  under the proportional rule. However, the payoffs under the equally gain payoff are  $(383\frac{2}{3}, 223\frac{2}{3}, 168\frac{2}{3})$ .

### 5.3.2 Stepwise expansion from singular coalitions to the grand coalition

In the preceding section singular coalitions merged to the grand coalition in one step. A generalization of this idea is *stepwise merging* of coalitions. In every step two coalitions merge until eventually the grand coalition  $N$  has been formed. If two coalitions, say  $S$  and  $K$ , merge then under the equal division of the *net gain value* of  $S$  and  $K$ ,  $ngv(S, K)$ , every player in  $S \cup K$  receives  $\frac{ngv(S, K)}{|K|+|S|}$ , and every player outside receives nothing. A merge is stable if for the disjoint coalitions  $S$  and  $K$ ,  $ngv(S, K) = v(S \cup K) - (v(S) + v(K)) \geq 0$ , i.e. if the game  $(N, v)$  is *superadditive*. Since convexity implies superadditivity, we conclude that such a merge is stable. The Proposition 5.3, therefore, is obtained.

**Proposition 5.3** *Under the equal division of the net gain value, each player is better off than in the original coalition.*

**Example 5.4** Consider the FG in example 4.1. The following table shows the payoff vectors for the various coalitions under the equally gain payoff.

Starting coalition	1	2	3	Sum	Loss <sup>1)</sup>	Gain <sup>2)</sup>
<sup>1)</sup> Without expanding (single)	336	176	121	633	143	-
{1,2}	336	176	121	633	143	-
{1,3}	$367\frac{1}{2}$	176	$152\frac{1}{2}$	696	80	-
{2,3}	336	188	133	657	119	-
<sup>2)</sup> With expanding (grand coalition)	$383\frac{2}{3}$	$223\frac{2}{3}$	$168\frac{2}{3}$	776	0	143
{1,2}-3	$383\frac{2}{3}$	$223\frac{2}{3}$	$168\frac{2}{3}$	776	0	143
{1,3}-2	$394\frac{1}{6}$	$202\frac{2}{3}$	$179\frac{1}{6}$	776	0	80
{2,3}-1	$375\frac{2}{3}$	$227\frac{2}{3}$	$172\frac{2}{3}$	776	0	119

<sup>1)</sup>Relative to the grand coalition.

<sup>2)</sup>Relative to the standing situation.

The upper panels shows the payoff for each country (single) coalition (first row) and for two-country coalitions<sup>20</sup>. The lower panel depicts the payoff under 2-step expansion. Consider for instance the second row {1,3}-2. In the first step

<sup>20</sup>Note that the Shapley value and the equally gain payoff coincide for every one and two-country coalition. However, for more-country coalitions this observation is no longer valid.

countries 1 and 3 merge, and the second step, coalition  $\{1,3\}$  and  $\{2\}$  merge. Initially, every country  $i$  receives its stand alone value  $v(\{i\})$ . Players 1 and 3 divide  $ngv(\{1\}, \{3\}) = v(\{1,3\}) - v(\{1\}) - v(\{3\}) = 63$ . In the second step coalition  $\{1,3\}$  and player 2 divide  $ngv(\{1,3\}, \{2\}) = v(\{1,2,3\}) - v(\{1,3\}) - v(\{2\}) = 80$ . This yields the following payoff scheme

coalitions	payoff of player 1	payoff of player 2	payoff of player 3
1-2-3	336	176	121
13-2	$367\frac{1}{2}$	176	$152\frac{1}{2}$
123	$394\frac{1}{6}$	$202\frac{2}{3}$	$179\frac{1}{6}$

Hence, if a coalition  $S = \{1,3\}$  does not accept player 2 to enter, coalition  $S$  will lose  $53\frac{1}{3}$ .

## 6 Concluding remarks

A basic issue in transboundary fishery management is the new entrant problem. It concerns the difficulty of negotiating mutually acceptable terms of entry that specify the petitioning nation's membership rights and obligations. The incumbent members face the likelihood of having to give up a portion of their present quotas to the newcomer. The applicant on the other hand believes that it may be better off to stay outside of the coalition.

In this paper we examined how a Regional Fishery Management Organization (RFMO) might achieve effective control of a high seas fishery. Starting point is that the countries who are charter members or potential entrants, have expressed an intention to exploit the stock in an efficient and sustainable way. We showed that the outcome of the non-cooperative solution is virtually identical to that of the unregulated open access fishery. When the combined harvesting efforts in the Nash equilibrium are larger than the carrying capacity the species will be depleted. Next we considered adjustment from the Nash equilibrium. In this regard we made use of the basic results that a fishery game has a unique Nash equilibrium under the competitive situation, and that it is convex under the cooperative situation. On the basis of these results we developed profit allocation schemes such that both the potential entrants and the charter members are better off than by staying out of the agreement. We proposed the proportional rule to achieve reduction of the harvesting level when countries act independently but have expressed an interest in reaching a sustainable or more profitable exploitation of the stock; the population monotonic allocation scheme as management rule for coalitions of various sizes, and the equal division of the net gain value to expand coalitions.

The above mentioned solutions are individually rational and efficient which are prerequisites for an agreement. This implies that application of the above mentioned management rules can lead to an arrangement that can be achieved without waiting time and transferable membership.

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**Appendix:** Calculation of the coalition value  $v_\alpha(S)$

Let  $(N, v)$  be a *CFG*. Assume that all fishing costs can be ordered in the following way:  $c_1 \leq c_2 \leq \dots \leq c_n$ . Following Proposition 4 in Norde *et al.* (2000) we have  $v(S) = \sum_{j \in S} f_{l_j}(b - c_j - l(N \setminus S) - 2l_{S,j})$  for every  $S \subseteq N$ . Here  $f_{l_j}$  is the  $C^1$ -function,  $f_{l_j} : \mathbb{R} \rightarrow \mathbb{R}$ , defined by

$$f_{l_j}(x) = \begin{cases} 0 & x \leq 0, \\ \frac{1}{4}x^2 & 0 < x \leq 2l_j, \\ l_j(x - l_j) & x > 2l_j. \end{cases}$$

and  $l_{S,j} = \sum_{k \in S; k < j} l_k$ .

For example, the values of coalitions  $\{1,3\}$  and  $\{1,2,3\}$  in Example 4.1 are calculated by  $v(13) = f_{14}(50) + f_{12}(8) = 504 + 16 = 520$ , and  $v(123) = f_{14}(58) + f_8(28) + f_{12}(0) = 776$ .